Internal Flow Characteristics of Simplex Swirl Atomizers

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The influences of main dimensions and operating conditions of simplex swirl atomizers on the thickness of the annular liquid film formed at the discharge orifice are examined. A general expression for film thickness is derived which, when compared with the experimental measurements reported in the literature, shows very satisfactory agreement over wide ranges of atomizer dimensions and liquid injection pressures. When measured values of Sauter mean diameter (SMD) are plotted against the corresponding values of liquid film thickness *t*, a relationship between SMD and *t* is obtained which corresponds closely to the results of previous studies on both airblast and simplex atomizers. The analysis also leads to the development of correlations for flow number and discharge coefficient in terms of the main nozzle dimensions. Predictions of flow number and discharge coefficient based on these equations show satisfactory agreement with the experimental results reported in the literature.

Nomenclature

A_a = air core area, m^2
A_o = discharge orifice area, m^2
A_p = total inlet ports area, m^2
C_D = discharge coefficient
D_p = inlet port diameter, m
D_s = swirl chamber diameter, m
d_o = discharge orifice diameter, m
FN = flow number, $\dot{m}/\sqrt{\Delta P \cdot \rho}$, m^2
Note: FN in (lb/h)/$\sqrt{\text{psi}}$ = FN in $m^2 \times 0.66 \cdot 10^6 \sqrt{\rho}$
K = nozzle constant, $(A_p/D_s d_o)$
K_v = velocity coefficient
L_p = length of inlet port, m
L_s = length of parallel portion of swirl chamber
ℓ_o = length of discharge orifice, m
$\dot{m}$ = liquid flow rate, kg/s
Q = volume flow rate, m^3/s
t = liquid film thichness, m
U = resultant velocity in orifice, m/s
U_{ax} = axial velocity in orifice, m/s
U_s = velocity at surface of liquid film, m/s
u = local flow velocity, m/s
X = ratio of air core area to orifice area, (A_a/A_o)
ΔP = injection pressure differential across nozzle, Pa
Δp = pressure drop along discharge orifice, Pa
θ = spray cone half-angle
μ = liquid viscosity, kg/ms
ρ = liquid density, kg/m^3

Introduction

THE simplest form of pressure swirl atomizer is the so-called "simplex" atomizer illustrated in Fig. 1. Fuel is fed into a swirl chamber through tangential ports that give the fuel a high angular velocity, thereby creating an air-cored vortex. The outlet from the swirl chamber is the final orifice, and the rotating liquid flows through this orifice under both axial and radial forces to emerge from the atomizer in the form of a hollow conical sheet. The actual cone angle is determined by the relative magnitude of the radial and axial components of velocity at exit. As the sheet expands, its thickness diminishes, and it soon becomes unstable and disintegrates into ligaments and then drops in the form of a well-defined, hollow cone spray.

Despite the geometrical simplicity of the simplex swirl atomizer, the hydrodynamic processes leading to the formation of a conical liquid sheet and to the disintegration of this sheet into ligaments and jets are highly complex. Previous studies have revealed the basic mechanisms involved in the conversion within the swirl chamber of a finite number of discrete liquid jets into a thin conical sheet, and have led to the formulation of quantitative relationships between the main atomizer dimensions and various flow parameters, such as discharge coefficient and initial spray cone angle. However, much less is known about the factors that govern the thickness of the sheet, or film, at the plane of discharge from the nozzle, although recent studies have shown a direct relationship between this initial film thickness and the mean drop size of the ensuing spray.[1,2]

Due primarily to their inherent simplicity, simplex atomizers have found wide ranges of engine and industrial applications. Therefore, it is of great practical interest to examine in detail the interrelationships that exist between nozzle design variables, internal flow characteristics, and various important spray features such as cone angle and mean drop size.

Film Thickness

The early studies of Taylor[3] and Giffen and Muraszew[4] led to the definition of an "atomizer constant" K, which is obtained as the ratio of the total area of the inlet ports to the product of the swirl chamber diameter and the discharge orifice diameter $(A_p/D_s d_o)$. They showed that any change in atomizer geometry which increases the atomizer constant will also yield a higher coefficient, a smaller spray cone angle, and a decrease in the ratio of air core area to discharge orifice area.

The influence of the initial thickness of the liquid sheet on spray characteristics produced by prefilming airblast atomizers was examined by Rizk and Lefebvre.[1] Their specially designed airblast atomizers were constructed to produce a flat liquid sheet across the centerline of a two-dimensional air duct, with the liquid sheet exposed to high velocity air on both sides. Analysis of the processes involved, and correlation of the experimental data, indicated that high values of liquid viscosity and liquid flow rate result in thicker films. It was also observed that thinner liquid films produce better atomization, according to the relationship SMD $\propto t^{0.4}$. This is an interesting

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result, since, if other parameters are held constant, the liquid film thickness is directly proportional to the nozzle size, which imples that the Sauter mean diameter (SMD) should be proportional to the 0.4 power of the atomizer linear scale. In fact, this is precisely the result obtained by El-Shanawany and Lefebvre[5] in their study of the effect of nozzle size on the SMD. Kutty et al.[6] have investigated the variation of spray cone angle and air core diameter with injection pressure differential. They measured air core size by taking photographs with the camera pointing upstream through the outlet orifice. Illumination was achieved by fitting a transparent window at the rear of the swirl chamber.[7] Their results showed that the liquid film thickness at the atomizer exit varies with nozzle operating conditions.

Simmons and Harding[8] derived a simple equation for liquid film thickness for simplex atomizers as a function of nozzle size and cone angle. However, this equation neglects any effects arising from variations in liquid properties and injection pressure differential.

Theory

Consider a small element of the liquid film flowing in the nozzle of length dx, depth dy, and a unit width in a direction at right angles to its direction of motion. It is assumed that only pressure and viscous forces act on this element, and under steady flow conditions these two forces balance each other, such that

$$\mathrm{d}p\cdot\mathrm{d}y=\mathrm{d}\tau\cdot\mathrm{d}x \tag{1}$$

where τ is the shear stress and dp represents the pressure gradient in the x direction. Since

$$\tau=\mu\frac{\mathrm{d}u}{\mathrm{d}y} \tag{2}$$

where μ is the liquid viscosity and u the velocity, then

$$\frac{\mathrm{d}p}{\mathrm{d}x}=\mu\frac{\mathrm{d}^2u}{\mathrm{d}y^2} \tag{3}$$

If the pressure gradient across the film is considered constant, Eq. (3) can be integrated to give

$$u=\frac{1}{\mu}\left(\frac{\mathrm{d}p}{\mathrm{d}x}\right)\frac{y^2}{2}+C_1y+C_2 \tag{4}$$

The constants C_1 and C_2 are determined from the boundary conditions, $y=0$, $u=0$, and $y=t$, $u=U_s$, where U_s is the maximum flow velocity attained at the liquid surface, and t the liquid film thickness. Hence,

$$U=\frac{1}{\mu}\frac{\mathrm{d}p}{\mathrm{d}x}\left(\frac{y^2}{2}-\frac{ty}{2}\right)+\frac{U_sy}{t} \tag{5}$$

It can be shown that the average velocity is given by

$$\bar{U}=-\frac{1}{\mu}\frac{\mathrm{d}p}{\mathrm{d}x}\frac{t^2}{12}+\frac{U_s}{2} \tag{6}$$

while the maximum velocity U_s is determined by differentiating Eq. (5) and equating to zero, to obtain

$$U_s=-\frac{1}{\mu}\frac{\mathrm{d}p}{\mathrm{d}x}\frac{t^2}{2} \tag{7}$$

Substituting for U_s from Eq. (7) into Eq. (6) gives the average flow velocity as

$$\bar{U}=-\frac{1}{\mu}\frac{\mathrm{d}p}{\mathrm{d}x}\frac{t^2}{3} \tag{8}$$

Due to the swirling motion created within the swirl chamber, the liquid has a flow direction that lies at an angle θ to a line drawn parallel to the orifice centerline. Substituting into Eq. (8) for dx = length of flow path, $l/\cos\theta$, and dp = pressure drop along this path Δp, leads to the following expression for the average flow velocity in the liquid film:

$$\bar{U}=\frac{\Delta pt^2\cos\theta}{3\mu l} \tag{9}$$

The fairly wide variations in internal geometry between various types of pressure swirl atomizers make it difficult to assign a value to l. However, all types of swirl atomizers, regardless of differences in internal geometry, invariably terminate in a circular discharge orifice, therefore, it is convenient to assume that l is proportional to d_o, i.e., $l=Ad_o$. Also, by assuming the pressure drop Δp along the flow path $l/\cos\theta$ is proportional to the total pressure drop across the nozzle, i.e., $\Delta p=B\Delta P$, then the average velocity at the nozzle exit is given by

$$\bar{U}=\frac{\Delta Pt^2\cos\theta}{3\mu d_o(A/B)} \tag{10}$$

Since the pressure drop is proportional to the flow length, the value of the ratio (A/B) should remain fairly constant and this is confirmed by experiment.

Any given particle of liquid leaves a point on the lip of the discharge orifice in a plane that lies tangential to the lip cylinder, at an angle which is the resultant of the axial and tangential components of velocity at this point. The included angle between the tangents to the spray envelope gives the cone angle, 2θ.

The axial component of velocity in the orifice is given by

$$U_{\mathrm{ax}}=\frac{\dot{m}}{\rho(A_o-A_a)}=\frac{\dot{m}}{\rho A_o(1-X)} \tag{11}$$

where $\dot{m}$ is the liquid flow rate, ρ the liquid density, A_o the orifice area, A_a the air core area, and $X=A_a/A_o$. Since $\cos\theta=U_{\mathrm{ax}}/\bar{U}$, by substituting for U_{ax} and $\bar{U}$ from Eqs. (11) and (10), respectively, leads to

$$\cos^2\theta=\frac{12\dot{m}\mu(A/B)}{\pi\rho d_o\Delta Pt^2(1-X)} \tag{12}$$

The experimental results reported in the literature support the validity of this expression, and indicate a best value of around 400 for A/B.

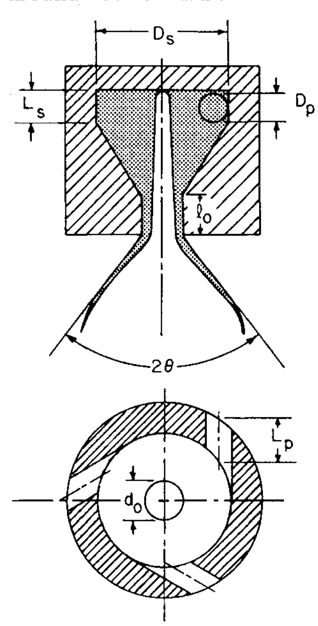

Fig. 1 Schematic view of a simplex swirl atomizer.

To obtain a more general equation for spray cone angle it is necessary to find a relationship between cone angle and other flow parameters, notably discharge coefficient. Now, Giffen and Muraszew[4] have shown that, if the size of the air core is such as to give maximum flow, the discharge coefficient is given by

$$C_D = \left[\frac{(1-X)^3}{(1+X)}\right]^{0.5} \tag{13}$$

where X may be derived directly from the nozzle constant K using the expression[4]:

$$2K^2X^2 = (1-X)^3 \tag{14}$$

Where comparisons have been made, it generally has been found that the measured values of C_D tend to exceed the theoretical predictions of Eqs. (13) and (14). However, if the actual measured values of X are substituted into Eq. (13) to determine X, rather than using Eq. (14), this equation then predicts values of C_D that are higher than the correponding experimental results. This can be explained as a result of friction losses which decreases both the tangential and axial velocities in the atomizer. The reduction in tangential velocity lowers the swirl and thickens the liquid film so that the effective flow area in the orifice increases. However, this beneficial effect on C_D is more than offset by the reduction in axial velocity, and so the coefficient of discharge is less that the theoretical value. To correct for this, a velocity coefficient can be introduced into Eq. (13) which takes account of the losses occurring in different parts of the nozzle, thus,

$$C_D = K_v \left[\frac{(1-X)^3}{(1+X)}\right]^{0.5} \tag{15}$$

Also, we have

$$\bar{U} = K_v[2\Delta P/\rho]^{0.5} \tag{16}$$

while the volume flow rate Q is given by

$$Q = A_o C_D [2\Delta P/\rho]^{0.5} = A_o C_D \bar{U}/K_v \tag{17}$$

Now, $\cos\theta = U_{ax}/\bar{U}$

Substituting for U_{ax} and $\bar{U}$ from Eqs. (11) and (17), respectively, gives

$$\cos\theta = \frac{C_D}{K_v(1-X)} \tag{18}$$

While, from Eqs. (15) and (18),

$$\cos^2\theta = \frac{1-X}{1+X} \tag{19}$$

We now have two equations for $\cos^2\theta$ namely, Eqs. (12) and (19). Equating these equations leads to the following expression for liquid film thickness:

$$t^2 = \frac{1560\dot{m}\mu}{\rho d_o \Delta P}\,\frac{(1+X)}{(1-X)^2} \tag{20}$$

or, by substituting for $\mathrm{FN} = \dot{m}/(\Delta P\rho)^{0.5}$,

$$t^2 = \frac{1560\mathrm{FN}\mu}{\rho^{0.5} d_o \Delta P^{0.5}}\,\frac{(1+X)}{(1-X)^2} \tag{21}$$

Since X is dependent on t, in fact, $X = A_a/A_o = (d_o - 2t)^2/d_o^2$, some trial and error procedures are involved in the solution of Eqs. (20) and (21).

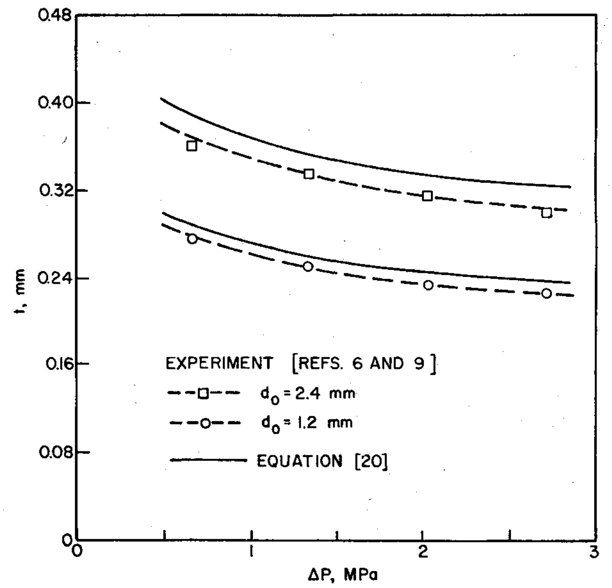

Fig. 2 Variation of film thickness with injection pressure for different orifice diameters.

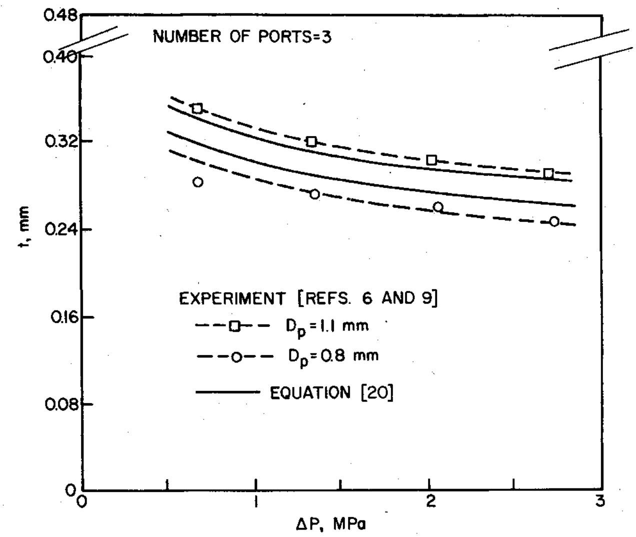

Fig. 3 Variation of film thickness with injection pressure for different inlet port diameters.

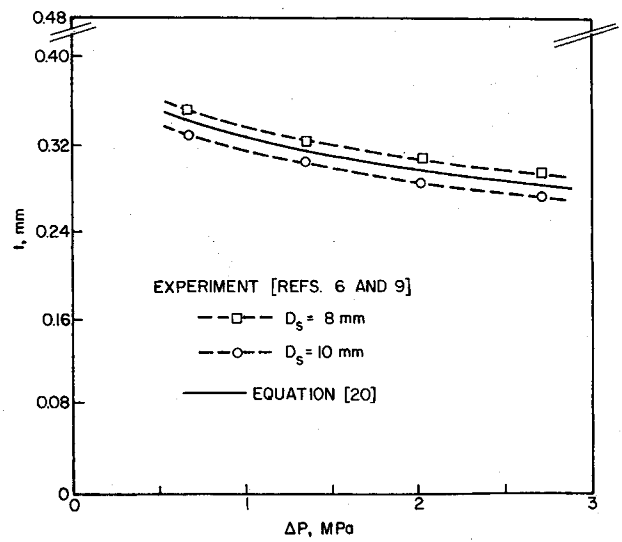

Fig. 4 Variation of film thickness with injection pressure for different swirl chamber diameters.

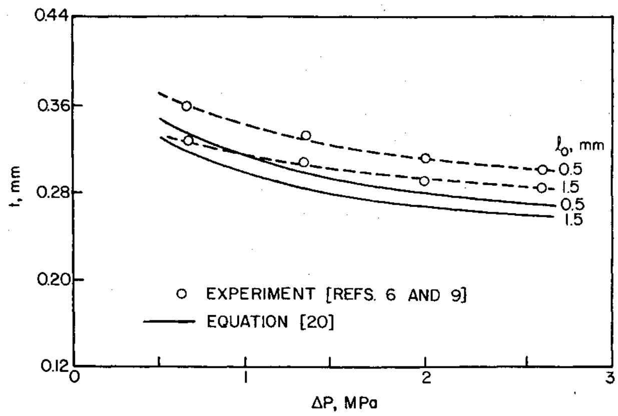

Fig. 5 Variation of film thickness with injection pressure for different orifice lengths.

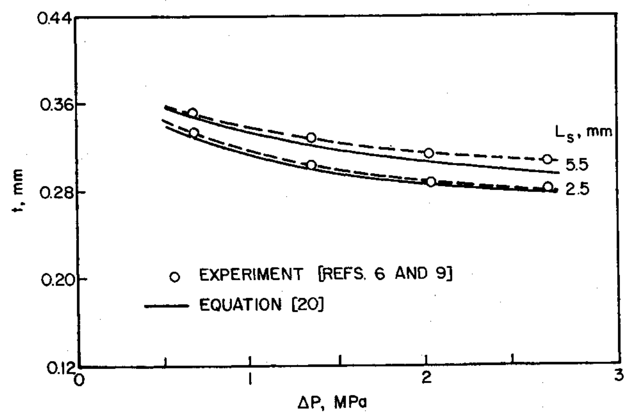

Fig. 6 Variation of film thickness with injection pressure for different swirl chamber lengths.

Equation (20) was employed to calculate film thickness for different nozzle dimensions and operating conditions, using the measured kerosine flow rate in each case.[6,9] The results are shown in Figs. 2-6 as plots of film thickness vs injection pressure differential. It is clear from these figures that a higher nozzle pressure drop produces a thinner liquid sheet. This may be readily explained by reference to Eq. (10) which shows that the resultant velocity at the nozzle exit is proportional to the product of pressure drop and the square of liquid film thickness. Now the continuity equation dictates that liquid velocity is proportional to liquid flow rate which, for any given atomizer, is proportional to the square root of nozzle pressure drop. Thus, from inspection of Eq. (10), it is clear that any increase in pressure drop must be accompanied by a reduction in film thickness. All of the experimental points in Fig. 2, corresponding to different nozzle sizes, confirm this trend of variation of film thickness with pressure drop.

The variation of film thickness with liquid ports area is shown in Fig. 3. Increasing the inlet area raises the flow rate through the nozzle, which results in a thicker film. The effect of a reduction in swirl chamber diameter is to increase the liquid film thickness, as shown in Fig. 4. This may be attributed directly to the lower swirl action which reduces the diameter of the core within the final discharge orifice. The effects of both orifice and swirl chamber lengths on film thickness are quite small, as illustrated in Figs. 5 and 6.

Measured values of Sauter mean diameter for kerosine are shown plotted against fuel injection pressure ΔP in Fig. 7. The different curves drawn in this figure correspond to different values of nozzle flow number. The substantial improvement in atomization quality with increase in ΔP is attributed not only to the higher discharge velocity, but also to the reduction in film thickness which accompanies an increase in ΔP.

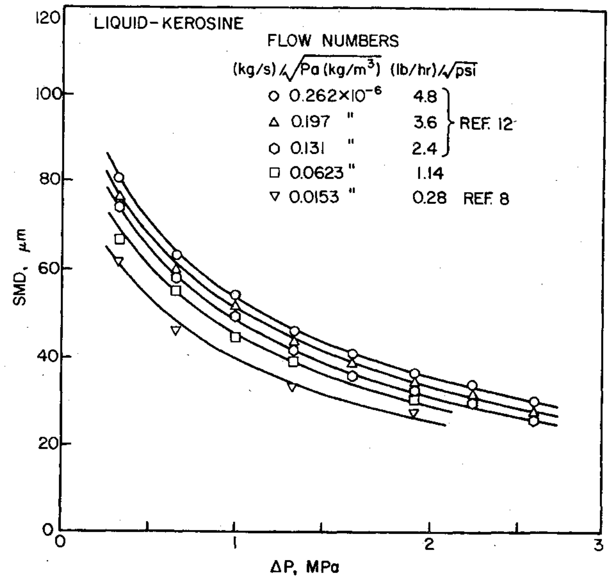

Fig. 7 Influence of injection pressure and flow number on Sauter mean diameter.

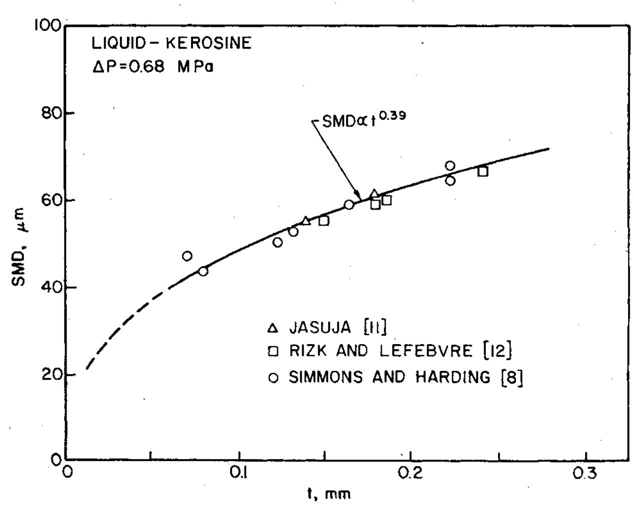

Fig. 8 Variation of Sauter mean diameter with liquid film thickness.

Equation (21) was used to determine values of liquid film thickness for all of the experimental points plotted in Fig. 7, as well as for other SMD data reported in Refs. 8 and 11. The results are shown plotted in Fig. 8, which indicate that SMD $\propto t^{0.39}$. This exponent of 0.39 is almost identical to that obtained by Rizk and Lefebvre[1] for prefilming airblast atomizers, and also agrees closely with the conclusion reached by Simmons.[2]

The results of the present study indicate that Eq. (21) could be simplified appreciably, while still retaining all the key parameters that have been shown to influence liquid film thickness, as follows:

$$t = 3.66\left[\frac{d_o \mathrm{FN} \mu}{(\rho \cdot \Delta P)^{0.5}}\right]^{0.25} \tag{22}$$

The above equation provides useful guidance on the effects of atomizer characteristics and liquid flow properties on film thickness and on the mean drop size of the ensuing spray. It is of interest to note that surface tension does not appear in the above expression for film thickness although, of course, it does play a major role in the subsequent breakup of the liquid sheet into ligaments and drops. The influence of liquid viscosity is clearly of major importance in the atomization process, because viscous forces impede atomization in two ways: 1) by

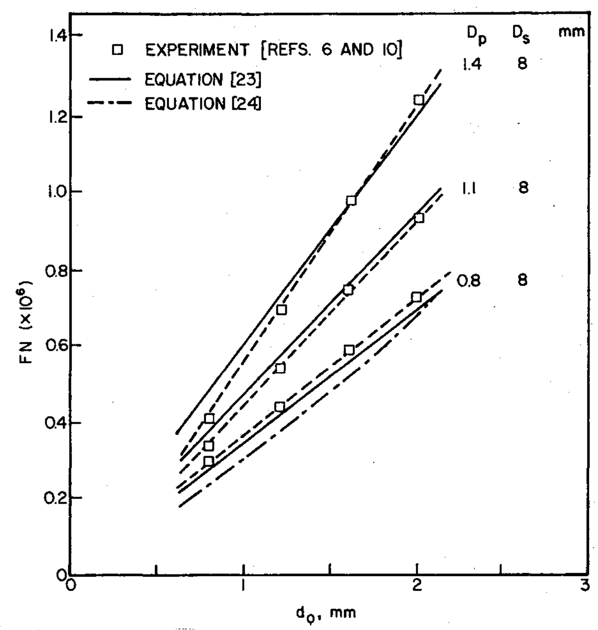

Fig. 9 Prediction of flow number for different orifice diameters.

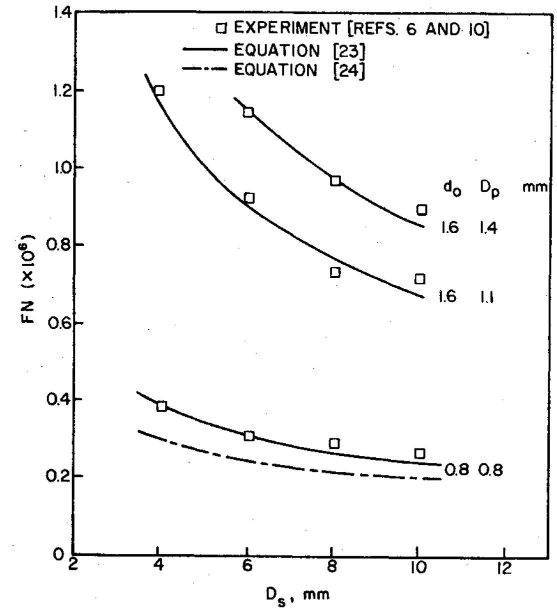

Fig. 11 Prediction of flow number for different swirl chamber diameters.

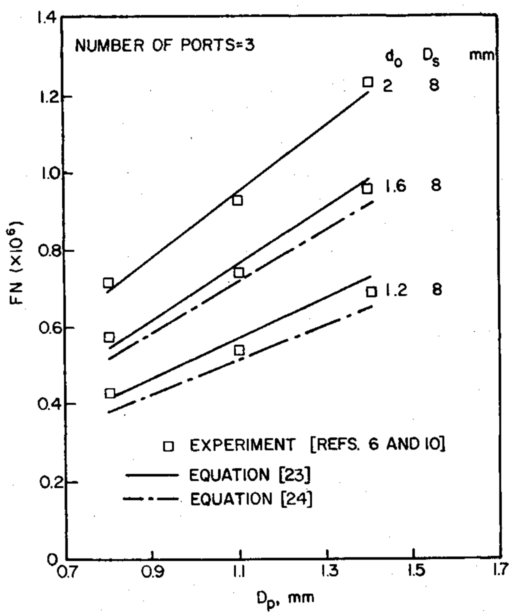

Fig. 10 Prediction of flow number for different inlet port diameters.

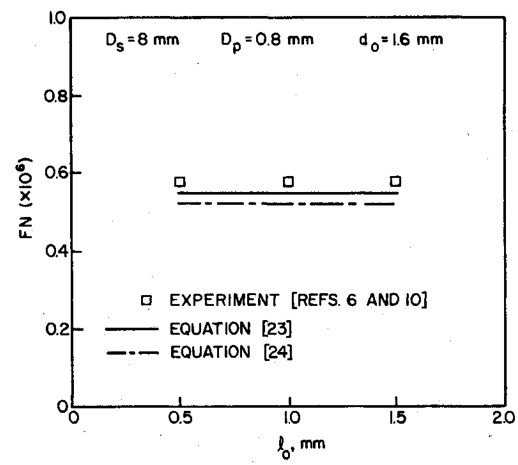

Fig. 12 Prediction of flow number for different orifice lengths.

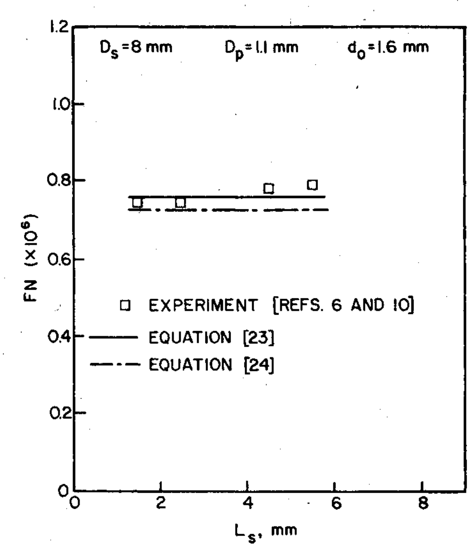

Fig. 13 Prediction of flow number for different swirl chamber lengths.

increasing the initial film thickness, and 2) by resisting the disintegration of the sheet into drops. Equation (22) shows that the effect of liquid density on film thickness is quite small ($t \propto \rho^{-0.125}$), thus, its influence on atomization quality should also be insignificant. This is confirmed by the results of measurements of spray quality on pressure-swirl atomizers.[13]

Flow Number

In the experimental studies conducted by Kutty et al.[6,10] a large number of simplex nozzles were manufactured such that the effect of variations in each of the key dimensions could be examined separately. Our analysis of their data leads to the following empirical expression for nozzle flow number:

$$FN = 0.0308\ [A_p^{0.5} d_o / D_s^{0.45}] \tag{23}$$

where A_p is the total area of the inlet ports in m^2, and d_o and D_s are the orifice and swirl chamber diameters, respectively, in m.

Figures 9-13 show the expected variation of flow number with changes in nozzle dimensions according to Eq. (23) along with experimental results from Refs. 6 and 10 for kerosine. These figures demonstrate the ability of Eq. (23) to predict nozzle flow number over wide ranges of atomizer dimensions.

Considering the various nozzle dimensions in Eq. (23), it is to be expected that higher values of discharge orifice diameter and inlet port area will increase the liquid flow rate for any given injection pressure differential, due to the increase in available flow area. The effect of increasing swirl chamber diameter in reducing flow rate, as indicated in Eq. (23) and demonstrated in Fig. 11, is due to the higher swirl action which enlarges the air core diameter and thereby reduces the effective flow area of the final orifice. The frictional losses in the nozzle are relatively small, so that the lengths of swirl chamber and final orifice have little effect on flow number.

Equation (23) can be modified to a dimensionally correct form as

$$FN = 0.395\,[A_p^{0.5} d_o^{1.25} / D_s^{0.25}] \tag{24}$$

Although this equation does not predict flow numbers to the same degree of accuracy as Eq. (23), the level of agreement between theory and experiment is still fairly close, as illustrated in Figs. 9-13.

Discharge Coefficient

The discharge coefficient of a swirl atomizer is inevitably low, due to the presence of the air core, which effectively blocks off the central portion of the orifice. According to the inviscid theory of Taylor,[3] the discharge coefficient is determined by the swirl-chamber geometry and is a unique function of K, the ratio of inlet port area to the product of swirl-chamber diameter and orifice diameter, $A_p/D_s d_o$. Other workers have also devised theories based on the assumption of idealized flow, but these theories have limited applications since, in practice, the use of viscous fluids modifies the flow patterns within the swirl chamber and causes significant deviations from the ideal case. For example, Giffen and Muraszew's[4] theoretical analysis resulted in expressions for the coefficient of discharge, the air core diameter, and the cone angle of the spray for an ideal (nonviscous) liquid in terms of the characteristic dimensions of the atomizer. However, when they compared their predictions with the results of measurements for actual liquids, some significant differences were observed. They concluded that the theoretical results should be regarded as indicating trends rather than giving numerical values.

Dombrowski and Hasson[14] demonstrated that correlations of discharge coefficient based on the atomizer constant K could be improved by plotting the measured values of discharge coefficient and cone angle against K multiplied by the ratio of swirl chamber diameter to orifice diameter, raised to some power.

An expression for discharge coefficient can be derived directly from Eq. (24) as

$$C_D = 0.35\,(A_p/D_s d_o)^{0.5}\,(D_s/d_o)^{0.25} \tag{25}$$

This relationship is very similar to that of Dombrowski and Hasson.[14] However, it leads to slightly higher values of C_D than the measured values reported in their study, as shown in Fig. 14.

Figure 15 illustrates the variation of air core area to orifice area ratio X with atomizer constant K at different levels of liquid injection pressure differential. The same area ratio is plotted against nozzle discharge coefficient in Fig. 16. The curves in Figs. 15 and 16 follow the same trends as the theoretical curves given in Ref. 4. However, they show that the relationships between X, K, and C_D are unique only for any given value of liquid injection pressure.

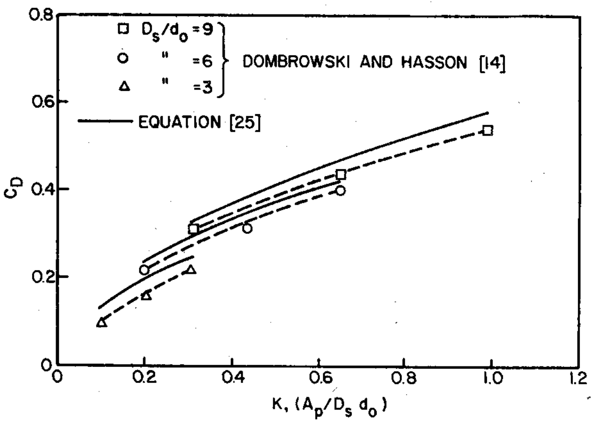

Fig. 14 Comparison between measured and prediction values of discharge coefficient.

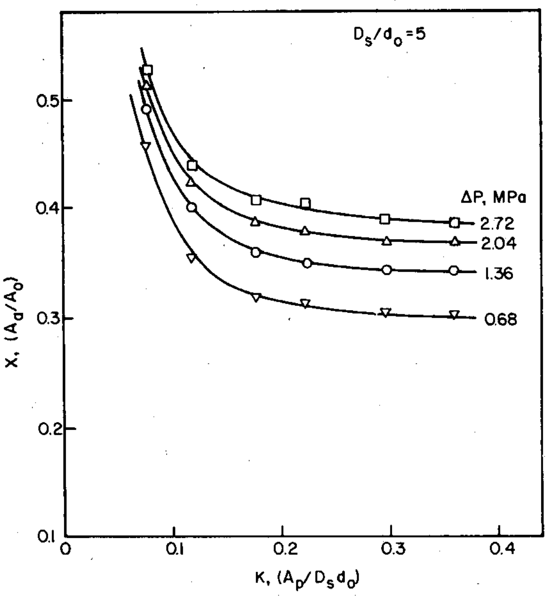

Fig. 15 Curves illustrating the dependence of the relationship between X and K on injection pressure.

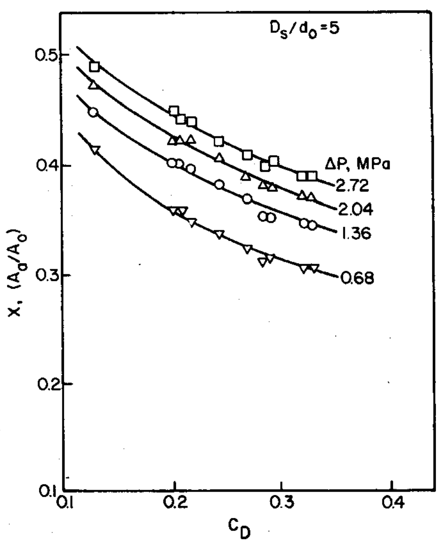

Fig. 16 Curves illustrating the relationship between C_D and X.

Conclusions

From analysis of the flow processes occurring in pressure-swirl atomizers an equation is derived for predicting the influence of the relevant atomizer design features and liquid flow properties on the thickness of the liquid film formed at the discharge orifice. This equation shows that the film thickens with increases in orifice diameter, nozzle flow number, and the absolute viscosity of the liquid. Thinner films are produced by an increase in liquid density and/or an increase in nozzle pressure drop.

The dependence of Sauter mean diameter on liquid film thickness is given by SMD $\propto t^{0.39}$, which is almost identical to the results reported previously for both simplex and airblast atomizers.

The expressions derived for flow number and discharge coefficient in terms of the main nozzle dimensions give results which agree closely with reported experimental values.

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